Lab Series LAB 6

EXERCISES FOR SECTION 4.6

In Exercises 1–10, use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the indicated value of n. Round your answer to four decimal places and compare the results with the exact value of the definite integral.

1. $\displaystyle\int_0^2 x^2\,dx,\quad n = 4$

2. $\displaystyle\int_0^1 \left(\frac{x^2}{2} + 1\right) dx,\quad n = 4$

3. $\displaystyle\int_0^2 x^3\,dx,\quad n = 4$

4. $\displaystyle\int_1^2 \frac{1}{x^2}\,dx,\quad n = 4$

5. $\displaystyle\int_0^2 x^3\,dx,\quad n = 8$

6. $\displaystyle\int_0^8 \sqrt[3]{x}\,dx,\quad n = 8$

7. $\displaystyle\int_4^9 \sqrt{x}\,dx,\quad n = 8$

8. $\displaystyle\int_1^3 (4 - x^2)\,dx,\quad n = 4$

9. $\displaystyle\int_1^2 \frac{1}{(x+1)^2}\,dx,\quad n = 4$

10. $\displaystyle\int_0^2 x\sqrt{x^2+1}\,dx,\quad n = 4$

In Exercises 11–20, approximate the definite integral using the Trapezoidal Rule and Simpson's Rule with $n = 4$. Compare these results with the approximation of the integral using a graphing utility.

11. $\displaystyle\int_0^2 \sqrt{1 + x^3}\,dx$

12. $\displaystyle\int_0^2 \frac{1}{\sqrt{1 + x^3}}\,dx$

13. $\displaystyle\int_0^1 \sqrt{x}\,\sqrt{1 - x}\,dx$

14. $\displaystyle\int_{\pi/2}^{\pi} \sqrt{x}\sin x\,dx$

15. $\displaystyle\int_0^{\sqrt{\pi/2}} \cos x^2\,dx$

16. $\displaystyle\int_0^{\sqrt{\pi/4}} \tan x^2\,dx$

17. $\displaystyle\int_1^{1.1} \sin x^2\,dx$

18. $\displaystyle\int_0^{\pi/2} \sqrt{1 + \cos^2 x}\,dx$

19. $\displaystyle\int_0^{\pi/4} x\tan x\,dx$

20. $\displaystyle\int_0^{\pi} f(x)\,dx,\quad f(x) = \begin{cases} \dfrac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}$

Getting at the Concept

21. If the function f is concave upward on the interval $[a, b]$, will the Trapezoidal Rule yield a result greater than or less than $\int_a^b f(x)\,dx$? Explain.

22. The Trapezoidal Rule and Simpson's Rule yield approximations of a definite integral $\int_a^b f(x)\,dx$ based on polynomial approximations of f. What degree polynomial is used for each?

In Exercises 23 and 24, use the error formulas in Theorem 4.19 to estimate the error in approximating the integral, with $n = 4$, using (a) the Trapezoidal Rule and (b) Simpson's Rule.

23. $\displaystyle\int_0^2 x^3\,dx$

24. $\displaystyle\int_0^1 \frac{1}{x+1}\,dx$

In Exercises 25 and 26, use the error formulas in Theorem 4.19 to find n such that the error in the approximation of the definite integral is less than 0.00001 using (a) the Trapezoidal Rule and (b) Simpson's Rule.

25. $\displaystyle\int_1^3 \frac{1}{x}\,dx$

26. $\displaystyle\int_0^1 \frac{1}{1 + x}\,dx$

In Exercises 27–30, use a computer algebra system and the error formulas to find n such that the error in the approximation of the definite integral is less than 0.00001 using (a) the Trapezoidal Rule and (b) Simpson's Rule.

27. $\displaystyle\int_0^2 \sqrt{1 + x}\,dx$

28. $\displaystyle\int_0^2 (x + 1)^{2/3}\,dx$

29. $\displaystyle\int_0^1 \tan x^2\,dx$

30. $\displaystyle\int_0^1 \sin x^2\,dx$

31. Prove that Simpson's Rule is exact when approximating the integral of a cubic polynomial function, and demonstrate the result for

$$\int_0^1 x^3\,dx,\quad n = 2.$$

32. Write a program for a graphing utility to approximate a definite integral using the Trapezoidal Rule and Simpson's Rule. Start with the program written in Section 4.3, Exercises 57–60, and note that the Trapezoidal Rule can be written as

$$T(n) = \tfrac{1}{2}[L(n) + R(n)]$$

and Simpson's Rule can be written as

$$S(n) = \tfrac{1}{3}[T(n/2) + 2M(n/2)].$$

[Recall that $L(n)$, $M(n)$, and $R(n)$ represent the Riemann sums using the left-hand endpoint, midpoint, and right-hand endpoint of subintervals of equal width.]

In Exercises 33–36, use the program in Exercise 32 to approximate the definite integral and complete the table.

n	$L(n)$	$M(n)$	$R(n)$	$T(n)$	$S(n)$
4					
8					
10					
12					
16					
20					

33. $\displaystyle\int_0^4 \sqrt{2 + 3x^2}\,dx$

34. $\displaystyle\int_0^1 \sqrt{1 - x^2}\,dx$

35. $\displaystyle\int_0^4 \sin\sqrt{x}\,dx$

36. $\displaystyle\int_1^2 \frac{\sin x}{x}\,dx$